Chapter 1 : Algebra
- ① Equation ✓ =
- ② Inequalities ✓ > ≥ < ≤
- ③ Complex no. ✓

① Equation
- ① Linear eq. x^1
- ② Quadratic eq. x^2
- ③ Other types of eq.
  - quadratic type
  - fractional
  - abs value
  - grouping

Linear eq.

Eg 1 : solve $6x - 7 = 2x + 5$

$4x = 12$

$x = 3$ *

Quadratic eq.
- ① Factorization * calculator.
- ② Completing the square (CTS)
- ③ Formula → (Decimal)

Eg 2 : Factorization

solve $6x(x-1) = 21 - x$

$6x^2 - 6x = 21 - x$

$6x^2 - 5x - 21 = 0$

$(3x-7)(2x+3) = 0$

$x = \frac{7}{3}$ $x = -\frac{3}{2}$

$(x-1)(x+2) = 0$

$x = 1$ $x = -2$

Tips!

RHS 0

↓

factorize

$(x-1.1)(x-1.23456) = 0$

$x = 1.1$ $x = 1.23456$

sq the 1st,
2xx,
sq the last

Completing the square (CTS)

expression

CTS: $x^2 + 4x - 1$

$= (x+2)^2 - 4 - 1$

$= (x+2)^2 - 5$ *

CTS: $x^2 - 2x + 2$

$= (x-1)^2 - 1 + 2$

$= (x-1)^2 + 1$ *

$x^2 = a$

$x^2 - (\sqrt{a})^2 = 0$

$(x+\sqrt{a})(x-\sqrt{a}) = 0$

$x = -\sqrt{a}$ $x = \sqrt{a}$

$x^2 = a$

$x = \pm\sqrt{a}$

$x^2 = 1$

$x = \pm 1$

$x^2 = 4$

$x = \pm\sqrt{4}$

$= \pm 2$

$x^2 = 3$

$x = \pm\sqrt{3}$

Eg 3 : CTS

Solve $x^2 - 5x + 3 = 0$

$$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 3 = 0$$

$$\left(x - \frac{5}{2}\right)^2 - \frac{13}{4} = 0$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$$

$$x - \frac{5}{2} = \pm\sqrt{\frac{13}{4}}$$

$$x - \frac{5}{2} = \pm\frac{\sqrt{13}}{2}$$

$$\therefore x = \frac{5}{2} \pm \frac{\sqrt{13}}{2} \#$$

$x^2 = \frac{25}{3}$

$x = \pm\sqrt{\frac{25}{3}}$

$= \pm\frac{\sqrt{25}}{\sqrt{3}}$

$= \pm\frac{5}{\sqrt{3}}$

CTS

a ≠ 1

expression

Factorize!

$3x^2 - 12x + 6$

$= 3\left[x^2 - 4x + 2\right]$

$= 3\left[(x-2)^2 - 4 + 2\right]$

$= 3\left[(x-2)^2 - 2\right]$

$= 3(x-2)^2 - 6$ ※

Eg 4 : CTS

equation

Solve $3x^2 - 12x + 6 = 0$ ÷3 both sides

$x^2 - 4x + 2 = 0$

$(x-2)^2 - 4 + 2 = 0$

$(x-2)^2 - 2 = 0$

$(x-2)^2 = 2$

$x - 2 = \pm\sqrt{2}$

$\therefore x = 2 \pm \sqrt{2}$ ※

Eg 5 : Formula

solve $\overset{a}{2}x^2 + \overset{b}{x} - \overset{c}{3} = 0$

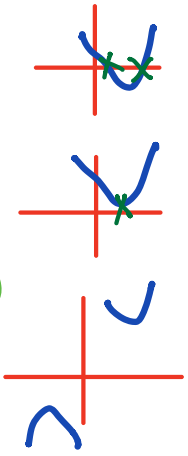
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

$b^2 - 4ac$

- > 0 (2 diff roots)
- $= 0$ (2 same roots / 1 root)
- < 0 (no real roots)

$\sqrt{-2}$ = no solution



$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-3)}}{2(2)}$$

$x = -\frac{3}{2}, 1$

$$= \frac{-1 \pm \sqrt{25}}{4}$$

$$= \frac{-1 \pm 5}{4}$$

$$= -\frac{6}{4}, 1$$

$$= -3/2, 1 \ \#$$

Eg 6:

Find the values of k for which the equation $(2k+3)x^2 + 2(k+3)x + (k+5) = 0$ has equal roots.

Also, solve the equation for those values of k.

($a = 2k+3$, $b = 2(k+3)$, $c = k+5$)

$$b^2 - 4ac = 0$$

$$(2k+6)^2 - 4(2k+3)(k+5) = 0$$

$$4k^2 + 24k + 36 - (2k+3)(4k+20) = 0$$

$$4k^2 + 24k + 36 - (8k^2 + 40k + 12k + 60) = 0$$

$$-4k^2 - 28k - 24 = 0 \quad \div -4$$

$$k^2 + 7k + 6 = 0$$

$$(k+6)(k+1) = 0$$

$$k = -6, -1 \quad \#$$

$k = -6$

$$-9x^2 - 6x - 1 = 0$$

$$9x^2 + 6x + 1 = 0$$

$$(3x+1)(3x+1) = 0$$

$$x = -\frac{1}{3} \quad \#$$

$k = -1$

$$x^2 + 4x + 4 = 0$$

$$(x+2)(x+2) = 0$$

$$x = -2 \quad \#$$

Tips!

① Domain: x exist!

Deno $\neq 0$

② Equate deno

③ multiply both sides

Other types of eq.

① Fractional eq.

Eg 7: solve $\frac{3x}{(x-2)} = \frac{1(x-2)}{1(x-2)} + \frac{6}{(x-2)}$

Domain

$x-2 \neq 0$

$x \neq 2$

$\frac{3x}{(x-2)} = \frac{x+4}{(x-2)} \times (x-2)$

$3x = x+4$

$2x = 4$

$x = 2$ ✗

No solution ※

Eg 8: solve $\frac{2x}{(x-3)} + \frac{5}{(x+3)} = \frac{36}{(x^2-9)}$

Domain

$x \neq 3, -3$

$$\frac{2x(x+3)}{(x-3)(x+3)} + \frac{5(x-3)}{(x+3)(x-3)} = \frac{36}{(x+3)(x-3)}$$

$\times (x+3)(x-3)$

$$\frac{2x^2+6x+5x-15}{(x+3)(x-3)} = \frac{36}{(x+3)(x-3)}$$

$$2x^2+11x-51 = 0$$

$$(2x+17)(x-3) = 0$$

$x = -\frac{17}{2}$ ✳ $\quad x = 3$ ✗

$\therefore x = -\frac{17}{2}$ only ✳

nth root $\sqrt[n]{x}$

② Radical

solve $\sqrt{5-x}+1=x-2$

$(\sqrt{5-x})^2=(x-3)^2$

$5-x=x^2-6x+9$

$x^2-5x+4=0$

$(x-1)(x-4)=0$

$x=1$ ✗ $x=4$ ✓

$\therefore x=4$ only

n=2 even Recheck!!

$\sqrt{5-x}+1=x-2$

Tips!

① $\sqrt[n]{x}=d$

1 side radical / : numbers root

② $(\sqrt[n]{x})^n=(d)^n$

n : even : Recheck !!

$(-2)^2=4$

$(2)^2=4$

n : odd : ~~Recheck?~~

$x=1$: $2+1=1-2$

$3\neq -1$ ✗

$x=4$: $1+1=4-2$ ✓

$2=2$ ✓

eg: $\sqrt[3]{x^2-1} = 2$

$n = 3$ odd

$\left(\sqrt[3]{x^2-1}\right)^3 = (2)^3$

$x^2 - 1 = 8$

$x^2 = 9$

$x = \pm\sqrt{9}$

$\therefore x = \pm 3$ ✳

$\sqrt{2x-3} - \sqrt{x+7} + 2 = 0$

Recheck? ✓ $\left(\sqrt{2x-3}\right)^2 = \left(\sqrt{x+7} - 2\right)^2$

$2x - 3 = x + 7 - 4\sqrt{x+7} + 4$

$(x - 14)^2 = \left(-4\sqrt{x+7}\right)^2$ (2)

$x^2 - 28x + 196 = 16(x+7)$

$x^2 - 44x + 84 = 0$

$(x - 42)(x - 2) = 0$

$x = 42$ ✗ $x = 2$ ✳

$\sqrt{2x-3} - \sqrt{x+7} + 2 = 0$

$x=42$: $9 - 7 + 2 = 0$, $4 \neq 0$ ✗

$x=2$: $1 - 3 + 2 = 0$, $0 = 0$ ✓

Power fraction / pelik²

3 terms

Quadratic type equation

① Factorize

② Middle term substitution

Let u : middle term

③ ( )² + u + = =

④ Back substitute

u = middle term

③ Rational exponents

solve $x^{4/3} - 5x^{2/3} + 6 = 0$

$(x^{\frac{2}{3}})^2 - 5(x^{2/3}) + 6 = 0$

let $u = x^{\frac{2}{3}}$

$u^2 - 5u + 6 = 0$

$(u-3)(u-2) = 0$

$u = 3 \qquad u = 2$

$(x^{2/3})^3 = (3)^3 \qquad (x^{2/3})^3 = (2)^3$

$x^2 = 27 \qquad x^2 = 8$

$\therefore x = \pm\sqrt{27}$ #

$x = \pm\sqrt{8}$

$= \pm\sqrt{4 \cdot 2}$

$= \pm 2\sqrt{2}$ #

$\sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

Solve $2x^{-2/3} - 7x^{-1/3} - 15 = 0$

$2\left(x^{-\frac{1}{3}}\right)^2 - 7\left(x^{-\frac{1}{3}}\right) - 15 = 0$

Let $u = x^{-\frac{1}{3}}$

$2u^2 - 7u - 15 = 0$

$(2u+3)(u-5) = 0$

$u = -\frac{3}{2} \qquad u = 5$

Back substitute

$\left(x^{-\frac{1}{3}}\right)^{-3} = \left(-\frac{3}{2}\right)^{-3} \qquad \left(x^{-\frac{1}{3}}\right)^{-3} = (5)^{-3}$

$x = -\frac{8}{27}$ # $\qquad x = \frac{1}{125}$ #

$$|a| = \begin{cases} a \text{ if } a \geq 0 \\ -a \text{ if } a < 0 \end{cases}$$

④ **Absolute value**

$|a| \begin{cases} +a \\ -a \end{cases}$

solve $|3x+5| = 1$

$3x+5 = \pm 1$

$3x+5 = 1$ $\qquad$ $3x+5 = -1$

$3x = -4$ $\qquad$ $3x = -6$

$x = -\frac{4}{3}$ ✱ $\qquad$ $x = -2$ ✱

Tips!

① One side = num

$|x| = a$

② $x = \pm a$

Factor by Grouping

4 terms

2 & 2 → variables.

3 & 1 $y^3 + x^2 + x + 1$

1 & 3 $x^2 + x + 1 - y^3$

⑤ Grouping

solve $x^3 + 2x^2 - x - 2 = 0$

$[x^3 + 2x^2] - [x + 2] = 0$

$x^2(x+2) - (x+2) = 0$

$a \quad b \quad - \quad b \quad = 0$

$(x+2)(x^2 - 1^2) = 0$

$(x+2)(x+1)(x-1) = 0$

$x = -2, \; x = -1, \; x = 1$ #

$ab - b = 0$

$b(a-1) = 0$

solve $3a^3 - 24a^2 - 12a + 96 = 0$

$[3a^3 - 24a^2] - [12a - 96] = 0$

$3a^2(a-8) - 12(a-8) = 0$

$x \quad y \quad - \quad 12 \quad y \quad = 0$

$(a-8)(3a^2-12) = 0$

$3(a-8)(a^2-4) = 0$

$3(a-8)(a+2)(a-2) = 0$

$a = 8$ * | $a = -2$ * | $a = 2$ *

H/W: $2x^4 + 8x^3 = 3x^2 + 12x$